HEM SHEELA MODEL SCHOOL
DURGAPUR
Mathematics Assignment
class X

Trigonometry

1. The shadow of a tower at a time is three times as long as its shadow when the angle of elevation of the sun is 60°.Find the angle of elevation of the sun at the time of the longer shadow. (Ans: 30°)

2. Prove that : $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta coesc\theta=1+\tan\theta+\cot\theta$

3. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long.Find the height of the tower. (Ans: 42 m)

4. The angle of elevation of the top of a tower at a point on the level ground is 30°.After walking a distance of 100 m towards the foot of the tower along the horizontal line through the foot of the tower on the same level ground, the angle of elevation of the top of the tower is 60°.Find the height of the tower. (Ans: 86.6 m)

5. If $x\sin^3\theta+y\cos^3\theta=\sin\theta\cos\theta$ and $x\sin\theta=y\cos\theta$,prove that $x^2+y^2=1$.

6. Find the value of $\frac{\sin^2 10^\circ+\sin^2 80^\circ}{\sec^2 20^\circ-\cot^2 70^\circ}-3\tan 80^\circ\tan 50^\circ\tan 45^\circ\tan 10^\circ\tan 40^\circ$.

7. Prove that : $(\sin A+\cos ecA)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$.

8. A person standing on the bank of a river observes that the angle of elevation of the top of a building standing on the opposite bank is 60°.When he moves 40 metres away from the bank, he finds the angle of elevation to be 30°.Find the eight of the building and the width of the river. (Ans: 20m,34.64m)

9. The angle of elevation of an aeroplane from a point A on the ground is 60°.After a flight of 30 seconds ,the angle of elevation changes to 30°.If the plane is flying at a constant height of 3600√3 m,find the speed in km/hour of the plane. (Ans: 864 km/hour)

10. If $\cot\theta=\frac{7}{8}$, evaluate: $i)\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$, $ii)\cot^2\theta$.

11. If $3\cot A=4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$ or not.

12. Prove that : $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\cos ecA+\cot A$

13. The angle of elevation of a jet plane from a point A on the ground is 60°.After a flight of 15 seconds ,the angle of elevation changes to 30°.If the jet plane is flying at a constant height of 1500√3m ,find the speed of the jet plane. (Ans: 720 km/h)

14. Without using trigonometric tables ,find the value of the expression :

$$\frac{\sec(90^\circ-\theta).\cos ec\theta-\tan(90^\circ-\theta)\cot\theta+\cos^2 25^\circ+\cos^2 65^\circ}{3\tan 27^\circ.\tan 63^\circ}$$

(Ans: 2/3)

15. Prove that : $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

16. If $\tan A = n\tan B$ and $\sin A = m\sin B$,prove that $\cos^2 A=\frac{m^2-1}{n^2-1}$.

17. Prove that: $(\cos ecA-\sin A)(\sec A-\cos A)=\frac{1}{\tan A+\cot A}$.

18. Prove that: $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}=\cot\theta$.

19. A kite is flying at a height of 60 m above the ground .The string attached to the kite is temporarily tied to a point on the ground .The inclination of the string with the ground is 60°.Find the length of the string ,assuming that there is no slack in the string. (Ans.- 40√3m)

20. A contractor plans to install two slides for the children to play in the park. For the children below the age of 5 years ,she prefers to have a slide whose top is at a height of 1.5m,and is inclined at an angle of 30°to the ground ,whereas for elder children ,she wants to have a steep slide at a height of 3 m ,and inclined at an angle of 60° to the ground .What should be the length of the slide in each case ? (Ans.- 3m,2√3m)

Mensuration

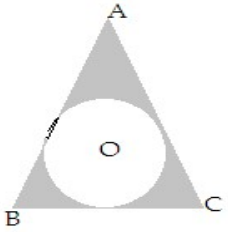
21. Two spheres of same metal weigh 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere .Find the diameter of the new sphere. (Ans: 12cm)

22. A hemispherical bowl of internal diameter 36 cm contains liquid .This liquid is filled into 72 cylindrical bottles of diameter 6 cm.Find the height of each bottle ,if 10%liquid is wasted in this transfer. (Ans: 5.4 cm)

23. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 4 km/h, in how much time will the tank be filled completely? (Ans: 5/4 h)

24. A metallic right circular cone 20 cm high whose vertical angle is 60°which is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire if diameter 1/16 cm ,find the length of the wire. (Ans: 7964.44m)

25. A right circular cone is divided into three parts by trisecting its height by two planes drawn parallel to the base .Show that the volumes of the three portions starting from the top are in the ratio 1:7:19.

26. 

A circle is inscribed in an equilateral triangle ABC of side 12 cm. Find the radius of the inscribed circle and the area of the shaded region. (Ans: 2√3cm, 24.6 sq.cm)

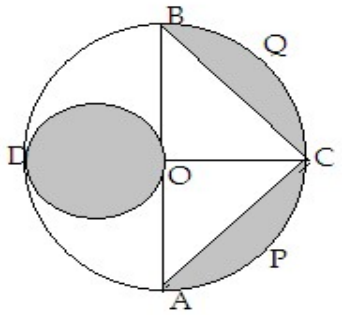
27. Two circles touch internally .The sum of their areas is 116 π cm^2 and distance between their centres is 6 cm.Find the radii of the circles. (Ans: 10 cm, 4cm)

28. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°.Find the total area cleaned at each sweep of the blades.
(Ans: 1254.96 cm^2)

29. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of the part of the field in which the horse can graze and also find the increase in the grazing area if the rope were 10 m long instead of 5m.
(Ans: 19.63m^2, 58.88m^2)

30. What is the area of the largest triangle that is inscribed in a semicircle of radius r unit?
(Ans: r^2sq.units)

31.



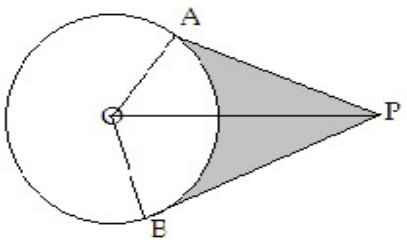
AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shade region. (Ans: 66.5 cm^2)

32. A toy is in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere .If the radius of base of the cone is 21cm and its volume is 2/3 of the volume of the hemisphere ,calculate the height of the cone and the surface area of the toy. (28 cm, 5082 cm^2)

33. A tent is in the shape of a right circular cylinder up to a height of 3 m and conical above it. The total height of the tent is 13.5m and radius of the base is 14 m. Find the cost of the cloth required to make the tent at the rate of Rs.80/m^2. (Ans: Rs. 82, 720)

34. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is 8/9 of the curved of the whole cone, find the ratio of the line segments into which the altitude of the cone is divided by the plane. (Ans: 1:2)

35.



An elastic belt is placed around the rim of a pulley of radius 5 cm.From one point C on the belt; the elastic belt is pulled directly away from the Centre O of the pulley until it is at P, 10 cm from the point Find the length of the belt that is still in contact with the pulley. Also find the shaded area.
(Ans.-20.93 cm, 17.08 cm^2)

Statistics and Probability

36. Calculate mean for the data given below:

Amount	Less than 100	Less than 200	Less than 300	Less than 400	Less than 500	Less than 600
No.of students	14	22	37	58	67	75

(Ans: 286)

37. Find the median of the following frequency distribution:

Marks	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
frequency	2	5	9	12	17	20	15	9	7	4

(Ans: 525)

38. The mean of the following frequency table is 53.But the frequencies F_1 and F_2 are missing. Find the missing frequencies.

Age(in years)	0-20	20-40	40-60	60-80	80-100	Total
No. of people	15	F_1	21	F_2	17	100

(Ans: F_1=18, F_2=29)

39. During the medical check-up of 35 students the weighs are recorded as follows

Weighs (in kg)	Less than 38	Less than 40	Less than 42	Less than 44	Less than 46	Less than 48	Less than 50	Less than 52
No. of students	0	3	5	9	14	28	32	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph.

40. Draw 'less than' ogive and 'more than' ogive for the following data and hence find its median.

Class	20-30	30-40	40-50	50-60	60-70	70-80	80-90
frequency	10	8	12	24	6	25	15

(Ans.-58.33)

41. The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50.Compute the missing frequencies :

Class	0-20	20-40	40-60	60-80	80-100	100-120	Total
Frequency	5	F_1	10	F_2	7	8	50

(Ans. F_1=8, F_2=12)

42. If the median of the distribution given below is 28.5, find the values of x and y.

Class	0-10	10-20	20-30	30-40	40-50	50-60	Total
frequency	5	x	20	15	y	5	60

(Ans: x=8, y=7)

43. A number x is selected at random from the numbers 1,2,3and 4.Another number y is selected at random from the numbers 1,4,9 and 16.Find the probability that product of x and y is less than 16. (Ans: 1/2)

44. Find the probability that in a leap year there will be 53 Tuesdays. (Ans: 2/7)

45. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992.What is the probability that the 2 students have the same birthday?

(Ans: 0.008)

46. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag .What is the probability that she takes out i) an orange flavoured candy? ii) a lemon flavoured candy? (Ans: 0,1)

47. A game consists of tossing a one-rupee coin 3 times and noting the outcome each time. Ramesh wins the game if all the tosses give the same result (i.e. three heads or three tails) and loses otherwise .Find the probability of Ramesh losing the game. (Ans: 3/4)

48. Five cards –the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

i) What is the probability that the card is the queen?

ii) If the queen is drawn and put aside, what is the probability that the second card picked up is a) an ace? b) a queen? (Ans: 1/5,1/4,0)

49. A letter is chosen at random from the letters of the word 'UNIVERSAL' .find the probability the letter chosen is not a vowel. (Ans: 5/9)

50. 1000 tickets of a lottery were sold and there are 5 prizes on these tickets .If Saket has purchased one lottery ticket, what is the probability of winning a prize? (Ans: 1/200)

Number System

51. Using Euclid's division lemma show that the cube of any positive integer is of the form 9m, 9m+1 or 9m+8 for some integer m.

52. Without actual division, show that the rational numbers are nonterminating repeating decimal

(i) $\frac{73}{2^2 \times 3^3 \times 5}$,

(ii) $\frac{129}{2^2 \times 5^3 \times 7^2}$.

53. Prove that $5\sqrt{2}+\sqrt{3}$ is irrational.

54. Using Euclid's division algorithm, find whether the pair of numbers 847, 2160 are co-prime or not.

[Ans: Yes, co-prime]

Algebra

55. If (–5) is a root of the equation $2x^2 + px - 15 = 0$ and the equation $p\left(x^2 + x\right) + k = 0$ has equal roots, then find the values of p and k. [Ans. 7, 7/4]

56. Find the zeros of the polynomial $f\left(x\right) = x^3 - 12x^2 + 39x - 28$, if it is given that the zeros are in AP. [Ans: 1,4,7 or 7,4,1]

57. If two zeros of the polynomial $x^4 + 3x^3 - 20x^2 - 6x + 36$ are $\sqrt{2}$ and $-\sqrt{2}$, find the other zeros of the polynomial. [Ans. 3 and –6]

58. Obtain the zeros of quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$ and verify the relation between its zeros and coefficients. $\left[\text{Ans: } 2\sqrt{3} \text{ and } 2/\sqrt{3}\right]$

59. If α, β are zeros of polynomial $6x^2 + x - 1$, then find the value of

(i) $\alpha^3\beta + \alpha\beta^3$, (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$ $\left[\text{Ans:(i) } \frac{-13}{216}, \text{(ii) } \frac{-2}{3}\right]$

60. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work. [Ans: 140 days, 280 days]

61. Form the pair of linear equations in this problem, and find its solution graphically: 10 students of class X took part in a Mathematics quiz. If the no. of girls is 4 more than the no. of boys, find the no. of boys and girls who took part in the quiz. [Ans: 3 and 7]

62. Find the values of k for which the system of equations $kx - y = 2, 6x - 2y = 3$ has (i) a unique solution, (ii) no solution, (iii) infinitely many solutions. [Ans: $k \neq 3, k = 3,$ no real value of k]

63. Solve for x and y: $\frac{xy}{x+y} = \frac{6}{5}, \frac{xy}{y-x} = 6 \left(x \neq 0, y \neq 0 \text{ and } x \neq y\right)$. [Ans: $x = 2, y = 3$]

64. A man invested an amount at 12% per annum simple interest and another at 10% per annum simple interest. He received an annual interest of Rs. 2600. But, if he had interchanged the amounts invested, he would have received Rs. 140 less. What amounts did he invest at the different rates? [Ans: Rs. 15000 at 12% and Rs. 8000 at 10%]

65. If the roots of the equation $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ac}x + b = 0$ are simultaneously real, then prove that $b^2 = ac$.

66. Solve: $\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 6 = 0, \; x \neq -1.$ [Ans: –3/2 or –2]

67. Solve: $4^{(x+1)} + 4^{(1-x)} = 10$. [Ans: 1/2 or –1/2]

68. A person on tour has Rs. 4200 for his expenses. If he extends his tour for 3 days, he has to cut down his daily expenses by Rs. 70. Find the original duration of the tour. [Ans: 12 days]

69. Some student planned a picnic. The total budget for food was Rs. 2000. But 5 students failed to attend the picnic and thus the cost for food for each student increased by Rs. 20. How many students attended the picnic and how much did each student pay for the food? [Ans: 20, Rs. 20]

70. How many terms of the AP 63, 60, 57, 54,… must be taken so that their sum is 693? Explain the double answer. [Ans: n = 21 or 22, 22^{nd} term is 0]

71. Find the middle term of the sequence formed by all natural numbers between 9 and 95, which leave a remainder 1 when divided by 3. Also find the sum of the numbers on both sides of the middle term separately. [Ans: 52, 1043]

72. If the sum of the first m terms of an AP be n and the sum of its first n terms be m then show that the sum of its first m+n terms is – (m+n).

73. If the sum of the first p terms of an AP is the same as the sum of its first q terms (where $p \neq q$) then Show that the sum of its first (p+q) terms is zero.

74. Which term of the AP 24, 21, 18, 15, … is the first negative term? What is the term? [Ans: 10^{th}, –3]

Coordinate Geometry

75. If the points A(–2, 1), B(a, b) and C(4, –1) are collinear and a – b = 1, find the values of a and b. [Ans: a = 1, b = 0]

76. The vertices of ΔABC are A(4, 6), B(1, 5) and C(7, 2). A line-segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC. [Ans: 5/6 sq. unit, 1 : 9]

77. Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, –2) and B(3, 7). [Ans: 2 : 9]

78. Find the area of the ΔABC with A(–1, 4) and the mid-points of sides through A being (2, –1) and (0, –1). [Ans: 12 sq. unit]

79. Two vertices of a ΔABC are given by A(6, 4) and B(–2, 2) and its centroid is G(3, 4). Find the coordinates of the third vertex C of ΔABC. [Ans: (5, 6)]

80. Find the centre of a circle passing through (6, –6), (3, –7) and (3, 3). [Ans: (3, –2)]

Geometry

81. Draw a ΔABC with side BC = 7 cm, $\angle B = 45^\circ$, $\angle A = 105^\circ$. Then, construct a triangle whose sides are 3/4 times the corresponding sides of ΔABC.

82. Draw a circle of radius 3 cm. Draw a tangent to the circle making an angle of 30° with a line passing through the centre.

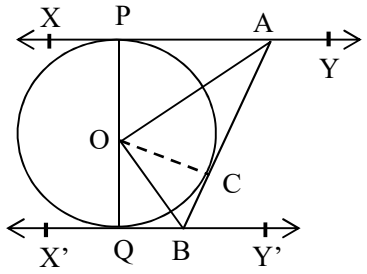
83. Draw a circle of radius 4.8 cm. Take a point P on it. Without using the centre of the circle, construct a tangent at the point P.

84. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

85. Draw two concentric circles of radii 4 cm and 6 cm. Construct a tangent to the smaller circle from a point on the larger circle. Verify the measurement by actual calculation. [Ans: 4.47 cm]

86. A circle is touching the side BC of ΔABC at P and touching AB and AC produced at Q and R respectively. Prove that AQ = ½(perimeter of ΔABC)

87. T is a point on the extended diameter BA of a circle with centre O. TP is a tangent to the circle from the external point T. If $\angle PBT = 30^\circ$, prove that BA : AT = 2 : 1

88. 

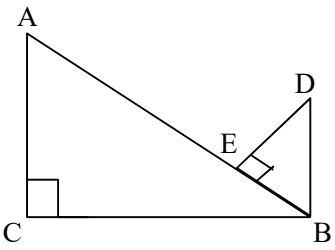
In the figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.

89. From an external point P, two tangents PA and PB are drawn to a circle C(O, r). If OP = 2r, show that ΔAPB is equilateral.

90. A quadrilateral ABCD is circumscribing a circle. Prove that AB + CD = AD + BC.

91. ΔABC is a right triangle in which $\angle C = 90^\circ$ and $CD \perp AB$. If BC = a, CA = b, AB = c and CD = p, then prove that (i) cp = ab, (ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

92. X and Y are two points on the sides AB and BC respectively, of ΔABC such that the line segment XY is parallel to the side AC and divides the triangle into two parts of equal area. Prove that $AX : AB = \left(2 - \sqrt{2}\right) : 2$.

93. 

In the given figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.

94. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm. find the corresponding side of the second triangle. [Ans: 5.4 cm]

95. ΔABC is an isosceles triangle in which AB = AC. E is a point on CB produced such that $EF \perp AC$. If $AD \perp CB$, prove that $AB \times EF = AD \times EC$.

96. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

97. ABC is a triangle in which AB = AC and D is a point on AC such that $BC^2 = AC \times CD$. Prove that BD = BC.

98. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

99. Two triangles BAC and BDC, right-angled at A and D are drawn on the same base BC and on the same side of BC. If AC and DB intersect at P, prove that $AP \times PC = DP \times PB$.

100. In ΔABC, D is the midpoint of BC and $AE \perp BC$. If $AC > AB$, show that

$$AB^2 = AD^2 - BC.DE + \frac{1}{4}BC^2.$$